The Regular Representation

$\langle I,(1,2),(1,3),(2,3),(1,2,3),(1,3,2)\rangle$

This is the space, a 6D space over GF(2)
Basis Vectors:

$\langle 1,0,0,0,0,0\rangle$

$\langle 0,1,0,0,0,0\rangle$

$\langle 0,0,1,0,0,0\rangle$

$\langle 0,0,0,1,0,0\rangle$

$\langle 0,0,0,0,1,0\rangle$

$\langle 0,0,0,0,0,1\rangle$

The regular representation of S_3.

$$\left\langle \begin{matrix} 1&0&0&0&0&0\\ 0&1&0&0&0&0\\ 0&0&1&0&0&0\\ 0&0&0&1&0&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&1 \end{matrix} \right\rangle \text{--I}$$

$$\left\langle \begin{matrix} 0&1&0&0&0&0\\ 1&0&0&0&0&0\\ 0&0&0&0&0&1\\ 0&0&0&0&1&0\\ 0&0&0&1&0&0\\ 0&0&1&0&0&0 \end{matrix} \right\rangle \text{--(1,2)}$$

$$\left\langle \begin{matrix} 0&0&1&0&0&0\\ 0&0&0&0&1&0\\ 1&0&0&0&0&0\\ 0&0&0&0&0&1\\ 0&1&0&0&0&0\\ 0&0&0&1&0&0 \end{matrix} \right\rangle \text{--(1,3)}$$

$$\left\langle \begin{matrix} 0&0&0&1&0&0 \\ 0&0&0&0&0&1 \\ 0&0&0&0&1&0 \\ 1&0&0&0&0&0 \\ 0&0&1&0&0&0 \\ 0&1&0&0&0&0 \end{matrix} \right\rangle \text{--(2,3)}$$

$$\left\langle \begin{matrix} 0&0&0&0&1&0 \\ 0&0&1&0&0&0 \\ 0&0&0&1&0&0 \\ 0&1&0&0&0&0 \\ 0&0&0&0&0&1 \\ 1&0&0&0&0&0 \end{matrix} \right\rangle \text{--(1,2,3)}$$

$$\left\langle \begin{matrix} 0&0&0&0&0&1 \\ 0&0&0&1&0&0 \\ 0&1&0&0&0&0 \\ 0&0&1&0&0&0 \\ 1&0&0&0&0&0 \\ 0&0&0&0&1&0 \end{matrix} \right\rangle \text{--(1,3,2)}$$

$$\langle I,(1,2),(1,3),(2,3),(1,2,3),(1,3,2)\rangle I =$$
$$\langle I,(1,2),(1,3),(2,3),(1,2,3),(1,3,2)\rangle$$
$$\langle I,(1,2),(1,3),(2,3),(1,2,3),(1,3,2)\rangle (1,2) =$$
$$\langle (1,2),I,(1,3,2),(1,2,3),(2,3),(1,3)\rangle$$
$$\langle I,(1,2),(1,3),(2,3),(1,2,3),(1,3,2)\rangle (1,3) =$$
$$\langle (1,3),(1,2,3),I,(1,3,2),(1,2),(2,3)\rangle$$
$$\langle I,(1,2),(1,3),(2,3),(1,2,3),(1,3,2)\rangle (2,3) =$$
$$\langle (2,3),(1,3,2),(1,2,3),I,(1,3),(1,2)\rangle$$
$$\langle I,(1,2),(1,3),(2,3),(1,2,3),(1,3,2)\rangle (1,2,3) =$$
$$\langle (1,2,3),(1,3),(2,3),(1,2),(1,3,2),I\rangle$$
$$\langle I,(1,2),(1,3),(2,3),(1,2,3),(1,3,2)\rangle (1,3,2) =$$
$$\langle (1,3,2),(2,3),(1,2),(1,3),I,(1,2,3)\rangle$$

Here's how this representation works. If G is any group, and $g \in G$, the function $f_g : G \to G$ given by $f_g(x) = xg$ for all $x \in G$, is a permutation of G. In other words, it is one-to-one and onto. If the order of G is denoted by $o(G)$ and $o(G) = n$, then $f_g \in S_n$. The regular representation of G is just the permutation matrices over F^n of the functions

f_g. In a sense, the regular representation of G is the group G itself, so G is its own representation. However don't be fooled by this glibness. The regular representation is a group of matrices, just like any other representation.

The reason the regular representation is used is because when you blow the representation up to this size so you have one dimension for each element of the group, you can prove lots of cool theorems. Like calculating the number of irreducible representations.