A New Kind of Reduction

Peter M. Maurer
Department of Computer Science
P.O. Box 97356
Baylor University
Waco, TX 76798-7356

Suppose we had a set of matrices with the following properties.

$$\begin{pmatrix} a & 0 & b & 0 \\ c & d & e & f \\ g & 0 & h & 0 \\ i & j & k & l \end{pmatrix}$$ If they are all like this, this constitutes a reduction!

Is this closed? Yes!

$$\begin{pmatrix} a_1 & 0 & b_1 & 0 \\ c_1 & d_1 & e_1 & f_1 \\ g_1 & 0 & h_1 & 0 \\ i_1 & j_1 & k_1 & l_1 \end{pmatrix}\begin{pmatrix} a_2 & 0 & b_2 & 0 \\ c_2 & d_2 & e_2 & f_2 \\ g_2 & 0 & h_2 & 0 \\ i_2 & j_2 & k_2 & l_2 \end{pmatrix} =$$

$$\begin{pmatrix} a_1a_2 + b_1g_2 & 0 & a_1b_2 + b_1h_2 & 0 \\ c_1a_2 + d_1c_2 + e_1g_2 + f_1i_2 & d_1d_2 + f_1j_2 & c_1b_2 + d_1e_2 + e_1h_2 + f_1k_2 & d_1f_2 + f_1l_2 \\ g_1a_2 + h_1g_2 & 0 & g_1b_2 + h_1h_2 & 0 \\ i_1a_2 + j_1c_2 + k_1g_2 + l_1i_2 & j_1d_2 + l_1j_2 & i_1b_2 + j_1e_2 + k_1h_2 + l_1k_2 & j_1f_2 + l_1l_2 \end{pmatrix}$$

Note that this is conjugate to a representation of the form $\begin{pmatrix} a & b & 0 & 0 \\ c & d & 0 & 0 \\ e & f & g & h \\ i & j & k & l \end{pmatrix}$

The multiplication also shows us where the new reductions are.

$$\begin{pmatrix} a & 0 & b & 0 \\ c & d & e & f \\ g & 0 & h & 0 \\ i & j & k & l \end{pmatrix} \Rightarrow \begin{pmatrix} a & b \\ g & h \end{pmatrix} \& \begin{pmatrix} d & f \\ j & l \end{pmatrix}$$

$\begin{pmatrix} c & e \\ i & k \end{pmatrix}$ is a function of the other two matrices.

This means, of course, that $\begin{pmatrix} a & 0 & b & 0 \\ 0 & d & 0 & f \\ g & 0 & h & 0 \\ 0 & j & 0 & l \end{pmatrix}$ is a decomposition.