

Note: Groups of size 120

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My program to classify subgroups of size 120 in $GL_5(2)$ has now completed. There are four non-conjugate classes of these groups. I do not yet know how many of these four are actually isomorphic to S_5 . I'm hoping the answer will be 1.

Originally I was enumerating every pair of generators from the set of all Order 2 and the set of all Order 5 matrices. This would have required something like 12 years of execution time (maybe as many as 20.) My algorithm to compare the generators instead of the generated groups, and use Gaussian elimination to determine the existence of a conjugacy matrix reduced the required time to about four weeks. However, this procedure also generated some anomalous results. I got 16 classes instead of four. It turns out, however, that this is due to something I didn't expect.

Let A be an order-5 generator of a group. It turns out that $A, A^2, A^3,$ and A^4 are pairwise non-conjugate, even though A can be replaced as a generator with $A^2, A^3,$ or A^4 without changing the generated group. I don't know if this is *always* the case (yet). But I know it's the case for at least one of the order 5 generators.